**Lecture 6 (Exotics Markets) Assignment**

Due start of class, Monday October 18, 2016

**Question 1 (6 marks)**

In class we looked at Gaussian copulas for pricing two-asset derivatives. We talked about using that model to price cross-pair options based on the two USD-pair option markets, calibrating the Gaussian copula correlation parameter such that the model reproduces the ATM cross option price.

When we do that, the model tends to underprice the butterfly – ie the implied volatility smile that comes out of the model is lower than the market. We said that was due to the fact that the model does not include any premium for stochastic correlation, which it should because correlation is not constant.

A stochastic correlation only affects the value of a derivative if its exposure to that correlation is non-linear. Let’s consider a specific case of the Gaussian copula to examine that: one where the RR and BF of the USD pairs is zero. In that case, their pricing is just Black-Scholes pricing, and if we assume a constant correlation, the pricing of the cross option is Black-Scholes with an implied volatility

Calculate the “gamma” of the cross option price with respect to the correlation parameter . Assume a market where the two USD-pair spots are 1, interest rates are zero, time to expiration is 0.5y, the USD-pair volatilities are both equal to 10%, and the correlation is +25%, and plot the correlation gamma as a function of strike for the cross-pair options.

Discuss the qualitative impact stochastic correlation should have on the cross-pair implied volatilities based on that plot.

**Question 2 (3 marks)**

Describe the market dynamic that is most important for knockout out pricing, and compare that to the market dynamic that is the most important for volatility swap pricing.

For each, explain why that market dynamic is important to the pricing.

**Question 3 (4 marks)**

Consider a dual digital option that pays $1 if EURUSD is above a strike K1 **and** GBPUSD is above a strike K2. All discount rates are zero. The price of the EURUSD European digital option (paying $1 if EURUSD is above K1) is 65% and the price of the GBPUSD European digital option (paying $1 if GBPUSD is above K2) is 30%.

Plot the price of the dual digital option priced under a Gaussian copula model, for correlation parameter ranging from -100% to +100%. Qualitatively explain the behavior of the price sensitivity to correlation.

Remembering that the main dynamic impacting the knockout price is risk reversal beta, qualitatively explain the behavior of the knockout price with .